International Journal of Heat and Mass Transfer 52 (2009) 289–300

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

On the intrinsic nature of jump coefficients at the interface between a porous medium and a free fluid region

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ARTICLE INFO

Article history:
Received 24 October 2007
Received in revised form 16 March 2008
Available online 4 August 2008

Keywords:
Porous media
Interface
Boundary conditions
Surface-excess quantities

ABSTRACT

In this paper, we discuss the physical nature of the jump parameters that generally appear in the expression for the jump conditions at a fluid/porous interface. These jump parameters are generally thought of as intrinsic interfacial properties, just like surface tension in the case of fluid/fluid interfaces. Based on a two-step up-scaling analysis, we show that jump parameters can be interpreted as surface-excess quantities. The value of a surface-excess quantity is shown to depend linearly on the position of the discontinuous interface and is therefore not an intrinsic property. We propose a theoretical approach that allows to introduce genuine intrinsic interfacial properties and to propose a best choice for the position of the discontinuous interface. We show that these properties are tightly related to the definition of the interfacial zone. This theoretical approach is successfully assessed on three important cases: a laminar flow parallel to a fluid/porous interface, a turbulent flow perpendicular to a porous/fluid interface and heat transfer perpendicular to a fluid/porous interface. It is believed that this approach is general enough to be applied to any interfacial transport phenomenon.

1. Introduction

In many applications, heat, mass and momentum transfer at the interface between a porous medium and an adjacent free fluid region play a key role in the process. These transfers are generally modeled through appropriate boundary conditions. The determination of the form of these boundary conditions is the subject of many scientific contributions and different methodologies are used to derive these boundary conditions. However, whatever the methodology used, so-called *jump coefficients* appear in the expression for these boundary conditions. For instance, in the case of the momentum transfer of a laminar flow parallel to the fluid/porous interface, Beavers and Joseph [2] introduced a jump coefficient α that relates the derivative of the velocity to the slip velocity at the interface:

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{K_p}}(u_B - U_D) \tag{1}$$

where u is the velocity in the direction tangential to the interface whose normal is in the y-direction, K_p is the permeability of the homogeneous porous medium, U_D is the Darcy velocity in the homogeneous porous medium and u_B is the slip velocity at the interface.

The theoretical determination of the value of these jump coefficients remains a scientific challenge as well as an important issue from a practical point of view [8,11]. These jump coefficients are very often interpreted or thought of as interfacial coefficients. In this regard, they are often considered as *intrinsic* interfacial properties. However, several authors noticed that these jump coefficients can depend on the location of the interface; this is in particular the case for the momentum transfer as pointed out in [13,14,20,21]. This observation tends to reconsider the interpretation of jump coefficients as genuine interfacial properties. This issue is actually related to the definition of the interface that separates the porous medium and the free fluid region. Indeed, by definition, an interface is a mathematical *surface of discontinuity*, possibly endowed with specific properties, called *interfacial properties*. Now, at the pore scale, the transition between the homogeneous porous medium and the free fluid region is all but sharp as illustrated in Fig. 1. Thus, it is a priori difficult or even impossible to define the position of the *equivalent* surface of discontinuity unambiguously [2]. Because of this ambiguous definition of the surface of discontinuity, the definition of the interfacial parameters is also ambiguous.

The issue we address in this paper is the interpretation of the jump coefficients that appear in the expression for boundary conditions at a fluid/porous interface. It must be emphasized that our goal is not to derive the form of these boundary conditions but is rather to study the intrinsic nature of the jump coefficients that necessarily appear in the expression for these boundary conditions. In this paper, several issues are addressed. What is the definition of a discontinuous fluid/porous interface? Should the value of the jump parameters depend on the location of the interface? Is it possible to define intrinsic interfacial properties?

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doi:10.1016/j.ijheatmasstransfer.2008.04.072

Nomenclature

c	mass fraction
D_h	hydraulic diameter
C_s	singular pressure drop coefficient
F	volumetric free energy
F^0	local contribution of the volumetric free energy
H^-, H^+	boundaries of the homogeneous regions
k	thermal conductivity
K	permeability
K_p	permeability of the homogeneous porous medium
p	pressure
S	mass source term
T	temperature
u	velocity
u_B	slip velocity
U_D	Darcy velocity
W	double-well volumetric free energy
y_M	location of the discontinuous interface
y_ψ	location of center of gravity of the variable ψ

Special symbols

$\tilde{\psi}_k$	ψ at the macroscopic scale in homogeneous region k

Greek symbols

α	slip coefficient of Beavers and Joseph [2]
μ^0	specific free enthalpy at saturation
δ	thickness of the interface region
η	continuous indicator function of the homogeneous regions
λ	capillary coefficient
ϕ	porosity
φ	conductive heat flux
ρ	fluid density

Superscripts

ϕ^{ex}	surface-excess quantity

Subscripts

ψ_+, ψ_-	constant asymptotic values of ψ outside the transition zone
ψ_e	double-well function related to ψ (see Eq. (11))

Following [5,6,16], our analysis is based on three different scales of description of the interface, as illustrated in Fig. 1. In particular, we introduce an intermediate level of description, denoted the *mesoscopic scale*. This latter scale of description is obtained after using an averaging volume that is the same as that used in the homogeneous porous medium to obtain the classical Darcy or Darcy–Brinkman equations for instance. It is worth noting that, unlike the case of homogeneous porous media where the validity of the closure relations can be proved under certain length-scale restrictions, up to now, no formal approach exists to derive the clo-

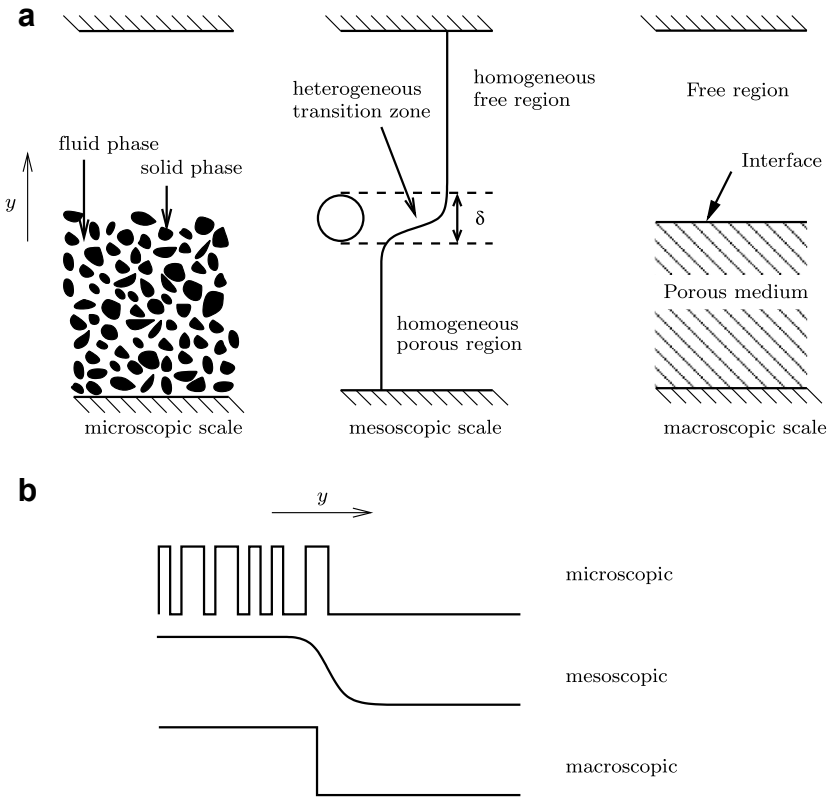

Fig. 1. The interface between a homogeneous porous medium and a free fluid region at different scales of description. (a) Fluid/porous interface at the three different scales. (b) Volumetric solid fraction at the three different scales.

sure relations within the interfacial region. Nevertheless, these closure relations can be postulated a priori and their validity can be checked a posteriori. For instance, the transport coefficients at the mesoscopic scale can be measured through volume-averaging where the filter size is generally the one adapted to the homogeneous porous region [3]. After such a volume-averaging procedure, the free-porous transition is a *continuous transition zone* across which all physical variables encounter strong but nevertheless continuous variations as illustrated in Fig. 1. The thickness δ of the interfacial transition zone is of the order of magnitude of the size of the averaging volume. The introduction of the intermediate mesoscopic scale might appear as a useless complex stage in the analysis. However, it has an important advantage: it allows to clearly dissociate the physical modeling part of the problem from the discussion on the interfacial properties. For the purpose of this paper, this allows to clarify the discussion on the nature of the interface and of the interfacial properties. Thus, in this paper, we focus on the second up-scaling, i.e., on the mesoscopic–macroscopic up-scaling. Indeed, in the first microscopic–mesoscopic up-scaling, no difficulty related to the nature of the interface exists: the main difficulty is related to the physical modeling of the system within the interfacial transition layer.

The paper is organized as follows. In Section 2, we remind the interpretation of a jump parameter as a surface-excess quantity and present some general features of such an interfacial property. Given this general presentation, in Section 3, we discuss the definition of an interfacial zone and introduce an intrinsic interfacial property, whose value is independent of the interface position. In Section 4, our general theoretical analysis is applied to three particular cases: a laminar flow parallel to the interface, a turbulent flow perpendicular to the interface and heat transfer perpendicular to the interface. In the three cases, our approach allows to determine (i) the corresponding boundary conditions and (ii) the intrinsic nature of the corresponding jump parameters.

2. General presentation

2.1. A simple example

In this section, we briefly present how the form of boundary conditions can be obtained by the analysis of interfacial processes [9]. For the sake of simplicity, we consider the following one-dimensional diffusion equation that is supposed to be valid at the mesoscopic scale:

$$\frac{\mathrm{d}^2 c}{\mathrm{d}y^2} = S(y) \quad \text{for } H^- < y < H^+$$

where c is the mass fraction of a given species, y is the coordinate normal to the interfacial region and $S(y)$ is a source term, which is supposed to reach significant values only within the interfacial zone as illustrated in Fig. 2; H^- and H^+ are the coordinates of the boundaries of the physical system, respectively, in the porous region and in the free fluid region. A similar diffusion reaction problem has been studied in [23].

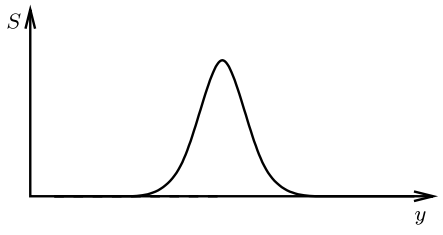

Fig. 2. Variation of the source term of mass across the interfacial zone.

At the macroscopic scale far from the interfacial zone, where $S = 0$, the following diffusion equation holds in each homogeneous region $k \in \{1;2\}$:

$$\frac{\mathrm{d}^2 \tilde{c}_k}{\mathrm{d}y^2} = 0 \tag{2}$$

where $\tilde{c}_k$ represents the mass fraction at the macroscopic scale in the region k. By construction of the macroscopic-scale model, it is not intended to capture small-scale variations. Thus, it is supposed that Eq. (2) hold up to the discontinuous interface.

By subtraction of the macroscopic and mesoscopic models in the entire domain, one gets:

$$\begin{cases} \frac{\mathrm{d}^2(c-\tilde{c}_1)}{\mathrm{d}y^2} = S(y) & \text{for } H^- < y < y_{\mathrm{M}} \\ \frac{\mathrm{d}^2(c-\tilde{c}_2)}{\mathrm{d}y^2} = S(y) & \text{for } y_{\mathrm{M}} < y < H^+ \end{cases} \tag{3}$$

where y_{M} is the location of the *equivalent surface of discontinuity at the macroscopic scale*. It must be emphasized that, in general, there is an infinite number of possible positions y_{M}. However, intuitively, the discontinuous interface should be located within the interfacial transition zone where $S(y)$ encounters strong variations. This issue will be discussed more thoroughly in the remainder of this article, but one must keep in mind that y_{M} is a priori defined with some arbitrariness and that one should seek for criteria to choose the most relevant interface position.

By integration of Eq. (3), first on $[H^-;\ y_{\mathrm{M}}]$ and then on $[y_{\mathrm{M}};\ H^+]$, one gets

$$\left.\frac{\mathrm{d}\tilde{c}_2}{\mathrm{d}y}\right|_{y_{\mathrm{M}}} - \left.\frac{\mathrm{d}\tilde{c}_1}{\mathrm{d}y}\right|_{y_{\mathrm{M}}} + \left.\frac{\mathrm{d}(c-\tilde{c}_2)}{\mathrm{d}y}\right|_{H^+} - \left.\frac{\mathrm{d}(c-\tilde{c}_1)}{\mathrm{d}y}\right|_{H^-} = \int_{H^-}^{y_{\mathrm{M}}} S(y)\,\mathrm{d}y + \int_{y_{\mathrm{M}}}^{H^+} S(y)\,\mathrm{d}y$$

The last two terms of the left-hand-side of the above equation vanish. Indeed, by construction, the macroscopic solution $\tilde{c}(y)$ is sought for so that it is equal to the mesoscopic solution $c(y)$ within each homogeneous region and in particular at their boundaries H^- and H^+. This yields:

$$\left.\frac{\mathrm{d}\tilde{c}_2}{\mathrm{d}y}\right|_{y_{\mathrm{M}}} - \left.\frac{\mathrm{d}\tilde{c}_1}{\mathrm{d}y}\right|_{y_{\mathrm{M}}} = \int_{H^-}^{H^+} S(y)\,\mathrm{d}y \tag{4}$$

Mathematically, this equation is a boundary condition at the interface located at y_{M} that couples the solutions of the macroscopic problem (i.e., $\tilde{c}_1(y)$ and $\tilde{c}_2(y)$). The right-hand-side is a so-called *surface-excess quantity*, denoted S^{ex}. The interfacial source of mass, that exists at the mesoscopic scale in the transition region, manifests itself at the macroscopic scale through this surface-excess source of mass. From a physical point of view, the boundary condition (4) is thus interpreted as an interfacial mass balance equation. This boundary condition is identical to that presented in [23].

The above analysis shows that, once the model at the mesoscopic scale has been determined, the form of the *corresponding* boundary condition at the interface can be determined following the steps of the above development. Edwards et al. [9] show that the above development is actually very general and can be applied to any kind of balance equation and to any kind of interface geometry (not necessarily planar as in our example). For the case of fluid/porous interfaces, it has already been successfully applied to laminar and turbulent flows [6,7].

It is worth noting that the meso–macro up-scaling presented in this example shows that no length-scale restriction is required to define the macroscopic model. Indeed, the only weak restriction lies in the fact that it must be possible to define outer regions (that can be small) where the macroscopic model holds. Once the macroscopic models and the corresponding outer regions can be de-

fined, jump conditions and corresponding jump parameters can be defined as well, without any length-scale restriction.

2.2. Surface-excess quantities

Jump parameters that appear in the expression for boundary conditions at a fluid/porous interface are generally interpreted, more or less wittingly, as interfacial physical properties, just like surface tension in the case of liquid/gas interfaces. In this section, we show that, in general, the value of a surface-excess quantity depends on the location of the interface y_M. Surface-excess quantities, and thus jump parameters, are therefore not intrinsic physical interfacial properties since their value depends on the *choice* of y_M.

Let us consider a volumetric physical variable ψ at the mesoscopic scale that varies continuously across the interfacial zone and reaches uniform values ψ^+ and ψ^- within the surrounding homogeneous regions as illustrated in Fig. 3(a). At this scale, the total amount of ψ in the system is

$$\Psi_{\text{meso}} = \int_{H^-}^{H^+} \psi(y)\,\mathrm{d}y$$

However, at the macroscopic scale, the variations of ψ within the interfacial zone are not captured and the interfacial zone is replaced by a discontinuous interface. Thus, at this scale, the total amount of ψ is

$$\Psi_{\text{macro}} = \int_{H^-}^{y_M} \psi^-\,\mathrm{d}y + \int_{y_M}^{H^+} \psi^+\,\mathrm{d}y$$

These two amounts are not necessarily equal. The difference $(\Psi_{\text{meso}} - \Psi_{\text{macro}})$ corresponds to the amount of ψ that is not accounted for by the macroscopic model and that must therefore be added so that the macroscopic and mesoscopic models are equivalent. This amount, denoted "surface-excess quantity", is assigned to the discontinuous interface (located at y_M); it is represented by the hatched area in Fig. 3(a):

$$\begin{aligned}\psi^{\text{ex}} &= \Psi_{\text{meso}} - \Psi_{\text{macro}} \\ &= \int_{H^-}^{y_M} (\psi(y) - \psi^-)\,\mathrm{d}y + \int_{y_M}^{H^+} (\psi(y) - \psi^+)\,\mathrm{d}y \end{aligned} \tag{5}$$

In three dimensions, a surface-excess quantity is a quantity per unit surface area assigned to the interface. For instance, in the example treated in Section 2.1, S^{ex} represents a mass source per unit surface area. It must be accounted for in the balance at the macroscopic scale to recover the mesoscopic balance:

$$\int_{V_1} \psi_1\,\mathrm{d}V + \int_{V_2} \psi_2\,\mathrm{d}V + \int_S \psi^{\text{ex}}\,\mathrm{d}S = \Psi_{\text{meso}}$$

where V_k is the volume of the homogeneous region k and S is the interface that separates the phases.

It is worth pointing out that the idea of assigning surface quantities at a fluid/porous interface has already been proposed in the literature and in particular in [15–17]. However, in these works, the authors treat both up-scaling steps without focusing in particular on the second one. However, it turns out that the second up-scaling step raises specific issues regarding the nature of the jump coefficients as shown in the remainder of this paper.

2.3. Variation of a surface-excess quantity with the position of the interface

Variations of the value of jump parameters on the location of the interface have been reported in the literature [3,13,20,21] and justified mathematically by Chandesris and Jamet [6]. In this section, we go back to this result and present it in a more general framework to explain this observed dependence using very simple arguments.

Graphically, Fig. 3(a) clearly shows that a surface-excess quantity ψ^{ex} depends on y_M. From Eq. (5), it is straigthforward to show that ψ^{ex} can be expressed as follows:

$$\psi^{\text{ex}}(y_M) = \int_{H^-}^{H^+} \psi(y)\,\mathrm{d}y - (\psi^+ H^+ - \psi^- H^-) + (\psi^+ - \psi^-) y_M \tag{6}$$

This expression shows that (i) ψ^{ex} depends linearly on y_M and that (ii) ψ^{ex} is independent of y_M when $\psi^+ = \psi^-$.

Thus, for any variable whose asymptotic values are different, one *should* expect a linear dependence of the jump parameter on the location of the interface. In the case of a laminar flow above a porous medium, Chandesris and Jamet [6] show that the jump

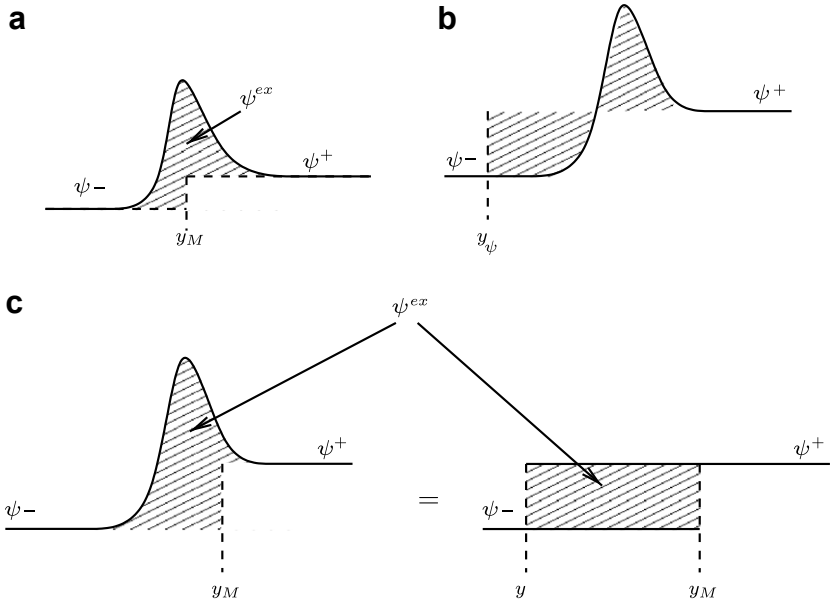

Fig. 3. Surface-excess quantity. (a) Graphical interpretation of ψ^{ex}. (b) Graphical interpretation of the center of gravity y_ψ. (c) Linear dependence of $\psi^{\text{ex}}(y_M)$.

parameter that appears in the boundary condition for the velocity at the interface is the surface-excess quantity $(\phi/K)^{ex}$, where ϕ is the porosity and K is the permeability. Fig. 7(a) clearly shows that the asymptotic values of $(\phi/K)(y)$ are different, which explains why, in this case, several authors observed a dependence of the jump parameter on the location of the interface.

In the general case where $\psi^+ \neq \psi^-$, the goal is to seek for true interfacial properties that are independent of the location of the interface. In this perspective, let us define the "center of gravity" y_ψ of the profile $\psi(y)$ defined such that

$$\int_{H^-}^{y_\psi} (\psi(y) - \psi^-)\mathrm{d}y + \int_{y_\psi}^{H^+} (\psi(y) - \psi^+)\mathrm{d}y = 0 \tag{7}$$

It can be shown that y_ψ exists if $\psi^+ \neq \psi^-$ (see Fig. 3)(b). We emphasize that y_ψ is an intrinsic property, in the sense that it depends only on the profile of the variable ψ across the interfacial zone and is thus independent of the position of the equivalent discontinuous interface y_M.

From Eq. (6) and (7), it is straightforward to express ψ^{ex} in the following compact form:

$$\psi^{ex}(y_M) = (\psi^+ - \psi^-)(y_M - y_\psi) \tag{8}$$

which is valid only if $\psi^+ \neq \psi^-$.

At this stage of the analysis, we showed that the value of a surface-excess quantity corresponding to a variable ψ such that $\psi^+ = \psi^-$ is independent of the position of the discontinuous interface and can be thus considered as a true interfacial property. However, if $\psi^+ \neq \psi^-$, we showed that the corresponding surface-excess quantity depends linearly on the position of the discontinuous interface y_M. We showed that, in this case, it is possible to define the center of gravity y_ψ whose value is independent of y_M. Nevertheless, we are still unable to provide a definition of a surface quantity that is independent of y_M and that could thus be considered as an interfacial property. This issue is tackled in Section 3.

The most widely known interfacial property is certainly surface tension. In the following section, we interpret surface tension as a surface-excess energy and show how the difficulty related to the interface position is solved. This analysis will be useful in the remainder of the article to propose an interpretation of interfacial quantities and a determination of intrinsic interfacial physical properties.

2.4. Surface tension: a surface-excess energy

In this section, we briefly remind how the van der Waals theory of capillarity allows to interpret surface tension as a surface-excess quantity (e.g. [19]). Surface tension is a macroscopic interfacial property coming from intermolecular forces at the microscopic scale where the particles are described individually. Using a simple mean-field approximation (e.g. [19]), it is possible to describe a liquid–vapor interface as a continuous transition zone where the fluid density varies smoothly (see Fig. 4)(a); this corresponds to the mesoscopic scale. At this scale, the volumetric free energy of the fluid is the following:

$$F = F^0(\rho) + \frac{\lambda}{2}(\nabla\rho)^2$$

where ρ is the fluid density, $F^0(\rho)$ is a non-convex function (see Fig. 4(b)) and λ is a constant coefficient, called the capillary coefficient. It can be shown that the bulk phase densities ρ_1 and ρ_2 are characteristic of the double-tangent of $F^0(\rho)$.

The excess free energy is given by

$$F^{ex} = F^{0^{ex}} + \left(\frac{\lambda}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}y}\right)^2\right)^{ex}$$

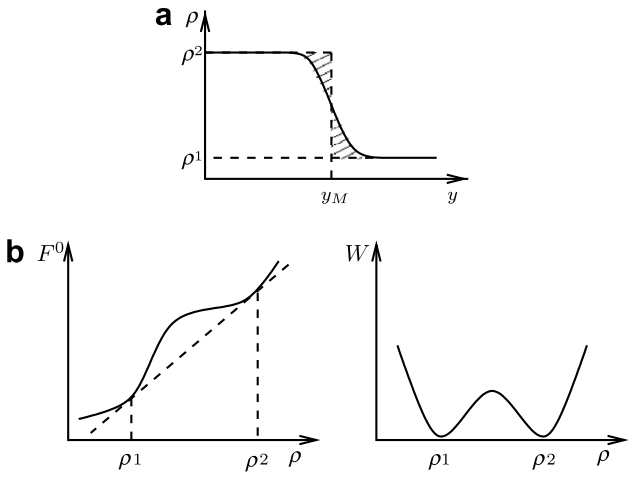

Fig. 4. Van der Waals model of capillarity: a diffuse interface model for liquid–vapor interfaces and surface tension. (a) Density profile across a liquid–vapor interface and its center of gravity. (b) Non-convex variation of the volumetric free energy F^0 (the slanting dashed line is the double-tangent) and of the corresponding double-well function W as a function of the density.

The last term does not depend on the position of the interface, whereas $F^{0^{ex}}$ does because $F^{0^+} = F^0(\rho_1) \neq F^0(\rho_2) = F^{0^-}$. Thus F^{ex} depends on y_M whereas surface tension does not. In the van der Waals theory, the function $W(\rho)$ that represents the difference between $F^0(\rho)$ and its double-tangent is then introduced (see Fig. 4(b))

$$\begin{aligned} F^0(\rho) &= W(\rho) + F^0(\rho_1) + \mu^0(\rho - \rho_1) \\ &= W(\rho) + F^0(\rho_2) + \mu^0(\rho - \rho_2) \end{aligned} \tag{9}$$

where μ^0 is the slope of the double-tangent. It is then straightforward to show that

$$F^{ex} = \underbrace{\mu^0\rho^{ex}}_{\text{variable part}} + \underbrace{W^{ex} + \left(\frac{\lambda}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}y}\right)^2\right)^{ex}}_{\text{intrinsic part (surface tension)}} \tag{10}$$

W^{ex} is independent of y_M because $W^+ = W(\rho_1) = W(\rho_2) = W^-$ $(= 0)$. Thus, the sum of the last two terms is independent of the position of the interface and represents surface tension.

It is important to note that, in this analysis, the density profile $\rho(y)$ defines the interfacial transition zone. Moreover, a natural way (often used) to define the position of the discontinuous interface y_M is to choose the center of gravity of the density profile: $y_M = y_\rho$. This choice ensures that the discontinuous interface is located within the interfacial zone, as illustrated in Fig. 4(a). For this particular location of the interface, the surface tension is exactly the surface-excess free energy (see Eq. (10)). This explains why this choice is generally made. It is also important to point out that the variable ρ is used to define the simple transformation $F^0(\rho) \to W(\rho)$ (see Eq. (9)) necessary to define an intrinsic surface energy. This simple transformation will be used in the analysis of interfacial properties in the case of fluid/porous interfaces to determine intrinsic interfacial quantities.

3. Interfacial zone and corresponding interfacial properties

When dealing with interfacial transport phenomena, the first issue deals with the definition of the interfacial zone. In some cases, this definition is unambiguous like for a liquid–vapor interface where the density profile $\rho(y)$ is naturally used. However, in other cases, not all the physical properties vary *exactly* at the same loca-

tions. For instance, at a fluid/porous interface, the typical porosity and permeability profiles obtained by filtering numerical results obtained at the microscopic scale [3] are represented in Fig. 7(a). In this figure, it is clear that the zones where the permeability and porosity profiles encounter strong variations are different. In this case, the definition of the interfacial transition zone is more ambiguous. However, intuitively, one would define the interfacial zone where both variables encounter strong variations. This issue is analyzed further in Section 3.2.

3.1. Interfacial properties: intrinsic and variable parts

Let us consider a physical variable ψ, whose asymptotic values are different in each homogeneous region (e.g. the permeability). Let η be a variable whose variation across the interfacial zone defines the interfacial zone and let δ be the thickness of this interfacial zone. As discussed above, this function is not always trivial to define because it cannot necessarily be one of the physical variables of the problem. The issue related to the construction of the function $\eta(y)$ is discussed in the next section. Here, we assume that the profile $\eta(y)$ has been determined and is thus considered as a given. The function $\eta(y)$ should be monotonic to ensure that it is a bijection. Typical profiles $\psi(y)$ and $\eta(y)$ are illustrated in Fig. 5(a). It is thus possible to define the function $\psi(\eta)$ as illustrated in Fig. 5(b).

From the function $\psi(\eta)$, it is now possible to define the function $\psi_e(\eta)$ as follows:

$$\psi_e(\eta) = \psi(\eta) - \left[\psi^- + \frac{\psi^+ - \psi^-}{\eta^+ - \eta^-}(\eta - \eta^-)\right] \tag{11}$$

This function, illustrated in Fig. 5(b), is the equivalent of the function $W(\rho)$ in the van der Waals model of capillarity. By construction, the asymptotic values of ψ_e are equal (and actually nil), so that ψ_e^{ex} *is independent of* y_M. ψ_e^{ex} can thus be considered as an interfacial property. It is worth noting that it has the features that one would expect for an interfacial property: (i) its value depends on the profile of the physical variable ψ to which it is associated and (ii) is associated to an interface defined through the function $\eta(y)$ that is supposed to be characteristic of the interfacial zone (and thus of the discontinuous interface). In the following, we show that both these properties can be made explicit in the expression for ψ_e^{ex} (cf., Eq. (15)).

From Eq. (11), it is straightforward to show that

$$\psi^{ex}(y_M) = \underbrace{\psi_e^{ex}}_{\text{Intrinsic}} + \underbrace{\frac{\psi^+ - \psi^-}{\eta^+ - \eta^-}\eta^{ex}(y_M)}_{\text{Variable}} \tag{12}$$

This equation is the equivalent of Eq. (10) in the van der Waals model of capillarity and shows that the only dependence on y_M of ψ^{ex} is actually the dependence of η^{ex}. Introducing the center of gravity y_η of the profile $\eta(y)$, one has

$$\psi^{ex}(y_M) = \psi_e^{ex} + (\psi^+ - \psi^-)(y_M - y_\eta) \tag{13}$$

By construction, the function $\eta(y)$ is such that it varies monotonically across the interfacial zone, and thus y_η is necessarily located within this interfacial zone, as illustrated in Fig. 5(a). In particular, the natural choice for the position of the equivalent interface is $y_M = y_\eta$ for which $\psi^{ex} = \psi_e^{ex}$ (see Eq. (13)). This means that, if the discontinuous interface is located at y_η, the surface-excess quantity assigned to this interface is the intrinsic interfacial property ψ_e^{ex}.

For some physical problems, such as a laminar flow above a homogeneous porous medium, it is known that it is important to account for the variation of the jump parameter with the interface position to recover the correct solutions in the adjacent homogeneous regions (namely the velocity profiles). However, for other physical problems, the issue of the variation of the jump parameter with the interface position does not seem to have been observed and addressed in the literature. For instance, for turbulent flows perpendicular to the interface, it is known that a singular pressure drop has to be accounted for (see Section 4.2.1). This pressure drop is generally considered as an interfacial property whose value is never assumed to be dependent on the position of the interface. However, from a modeling view point, no difference exists between these problems.

Eq. (13) shows that a surface-excess quantity is made of two parts: an intrinsic part ψ_e^{ex} that is independent of the interface position y_M and a variable part $(\psi^+ - \psi^-)(y_M - y_\eta)$ that depends linearly on y_M. If the variable part is small compared to the intrinsic part, then, whatever the interface position y_M within the transition zone, the value of the surface-excess quantity ψ^{ex} is almost equal to its intrinsic part ψ_e^{ex} (see Fig. 6(a)). In this case, no significant variation of the solution of the problem in the homogeneous regions is observed as the interface position y_M varies. However, when the variable part of the surface-excess quantity is of the same order of magnitude as its intrinsic part (see Fig. 6(b)), as the interface position y_M varies within the interfacial transition zone, the solution of the problem in the homogeneous regions may vary significantly depending on the position of the discontinuous interface. In this case, it is important to define the position of the interface and the corresponding surface-excess quantity precisely to recover the correct solution. Thus, Eq. (13) shows that the variable part of ψ^{ex} is weak compared to its intrinsic part provided that the following condition is satisfied:

$$|\psi_e^{ex}| \gg |\psi^+ - \psi^-|\delta \tag{14}$$

where δ is the interface thickness. This condition is illustrated in Fig. 6(a). In this case, the excess quantity ψ^{ex} can be considered as an intrinsic interfacial property. We can also define a more qualitative criterion: the variable part is weak if $(\max|\psi - \psi^-| \gg |\psi^+ - \psi^-|)$.

From Eqs. (8) and (13), it is easy to show that

$$\psi_e^{ex} = (\psi^+ - \psi^-)(y_\eta - y_\psi) \tag{15}$$

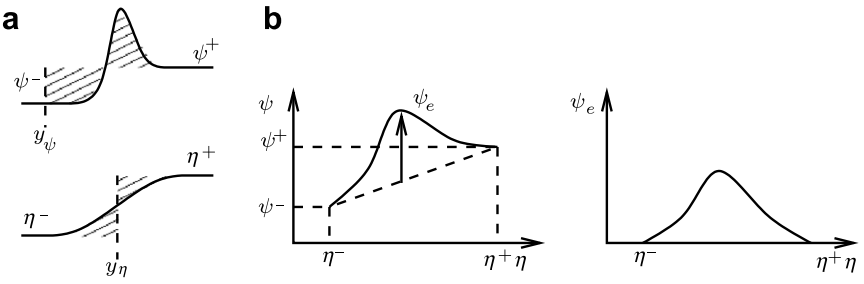

Fig. 5. Definition of the function $\psi_e(\eta)$. (a) $\psi(y)$ and $\eta(y)$. (b) $\psi(\eta)$ and $\psi_e(\eta)$.

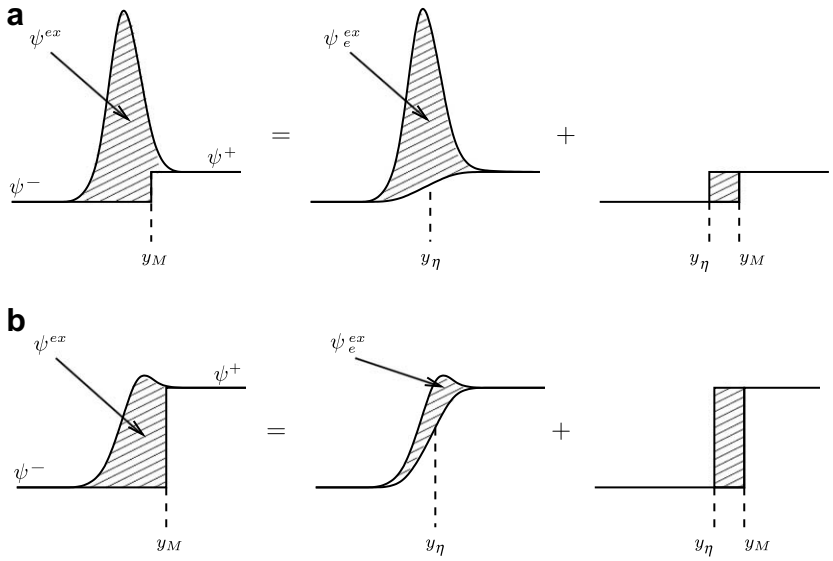

Fig. 6. Illustration of the relative importance of the intrinsic and variable parts of a surface-excess quantity. The curve below the curve $\psi(y)$ is $[\psi^- + (\psi^+ - \psi^-)(\eta(y) - \eta^-)/(\eta^+ - \eta^-)]$ (cf., Eq. (11)). (a) Weak variable part. (b) Large variable part.

so that the condition (14) can be written as follows:

$$|y_\eta - y_\psi| \gg \delta \tag{16}$$

The three geometrical parameters appearing in the above relation can easily be determined from the profiles $\psi(y)$ and $\eta(y)$.

Hitherto, we assumed that there exists a parameter η whose profile across the interface is characteristic of the interfacial transition zone. We showed that this parameter is important in particular to define the function $\psi(\eta)$ from which it is possible to define the intrinsic surface-excess quantity ψ_e^{ex}. In the following section, we address the issue of the determination of such a parameter η.

3.2. Dependence of an interfacial property on the definition of the interfacial zone

In the case of a laminar flow above a fluid/porous interface, we already pointed out that neither the porosity profile $\phi(y)$ nor the permeability profile $K(y) = \psi^{-1}(y)$ can be considered to define the interfacial zone. The interfacial zone should comprise the variations of *both* the porosity and permeability profiles. This choice is rather natural. However, in other cases, as turbulent flows perpendicular to a fluid/porous interface, the definition of the interfacial zone is more clearly a modeling choice. Indeed, numerical simulations using the $k-\epsilon$ model show that the production of turbulent kinetic energy due to the presence of the fluid/porous interface extends much further downstream than the interface thickness based on the porosity profile (see Section 4.2). In this case, the modeling choice is the following: either it is assumed that all this production is an interfacial feature, or it can be assumed that only part of this production is an interfacial feature and that the other part of the production is due to other physical mechanisms such as relaxation of turbulence that must be captured by the macroscopic model. For instance, in [4], the latter choice is made because the characteristic lengths of the homogeneous regions are of the same order of magnitude as the "turbulence-relaxation" zone, which means that the macroscopic model should capture the turbulence relaxation. However, if the characteristic length scale of the outer regions are much larger than the turbulence-relaxation zone, the macroscopic model should not be intended to capture this zone and the turbulence relaxation should be considered as an interfacial physical feature. This latter choice is made in Section 4.2. The introduction of the function η makes this modeling choice explicit.

We thus propose to introduce a new variable η, whose profile $\eta(y)$ is such that its thickness of variation comprises *all* the variations of the physical variables that the physicist interprets as interfacial characteristics. Hitherto, the only requirement to define the profile $\eta(y)$ is its thickness of variation δ. It is clear that many profiles satisfy this requirement as illustrated in Fig. 7(b) and the variable η is thus not unique: its choice is somewhat arbitrary. However, Eq. (15) shows that the value of the interfacial property ψ_e^{ex} depends very weakly on the choice of the function $\eta(y)$ since it actually depends *only* on the value of its center of gravity y_η and not on its entire profile (once δ has been chosen). Therefore, for a given range of variation and for a particular class of functions that are anti-symmetric from their inflexion point, the center of gravity of these functions are equal. Since this class is generally chosen for their simplicity, the center of gravity y_η is in practice almost unique (once the range of variation of the interface indicator function and thus the modeling choice has been made).

4. Examples of applications

In this section, the theoretical issues discussed in the previous sections are applied to three different cases to determine, in each case, the size of the interfacial zone and the corresponding intrinsic and variable parts of the corresponding jump parameters.

4.1. Laminar flow parallel to the fluid/porous interface

For a laminar flow parallel to a fluid/porous interface, Chandesris and Jamet [6] show that the relevant parameter that appears in the expression for the boundary condition is $(\phi/K)^{ex}$. In the following, we analyze the characteristics of this non-dimensional parameter, that is denoted ψ and defined by

$$\psi(y) = K_p \frac{\phi(y)}{K(y)}$$

Filtering numerical results obtained at the microscopic scale allows to determine the porosity and permeability profiles shown in Fig. 7(a) (e.g. [3]). It is worth mentioning that the interface thickness is directly related to the filter size, which is, to some extent, arbitrary. Here, the microscopic geometry is made of cubes regu-

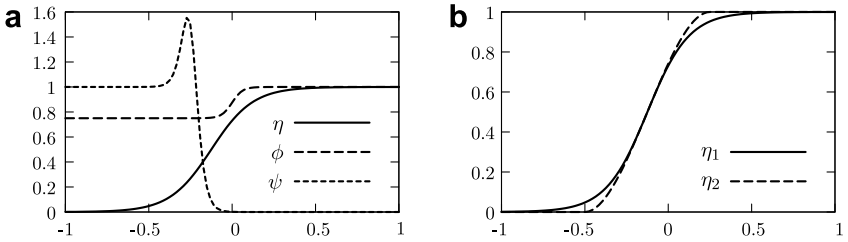

Fig. 7. Profile $\eta(y)$ whose variation defines the interfacial transition. It is such that its variations comprises the variations of *both* the porosity ϕ and permeability $K = \psi^{-1}$. (a) Example of profiles of physical quantities ϕ and ψ and of an interface indicator function η. (b) Different interface indicator functions: $\eta_1(y)$ is a hyperbolic tangent and $\eta_2(y)$ is a truncated polynomial of degree 3.

larly spaced and we used the cellular filter [18]. Fig. 7(a) shows that $\max(\psi - \psi^-) < (\psi^- - \psi^+)$. We showed in the previous analysis that, with this qualitative condition, one should expect the results obtained at the macroscopic scale to highly depend on the location of the discontinuous interface.

Quantitatively, we choose the profile $\eta(y)$ as a hyperbolic tangent that varies from 0 to 1 and whose thickness of variation is such that is comprises the thicknesses of variation of *both* profiles $\phi(y)$ and $\psi(y)$. The interfacial thickness is thus $\delta \simeq 0.2$. From the profiles $\phi(y)$, $\psi(y)$ and $\eta(y)$ of Fig. 7(a), it is found that their respective centers of gravity are $y_\phi = -0.09$, $y_\psi = -0.155$ and $y_\eta = -0.175$. Using Eq. (15), it is found that $\psi^{\mathrm{ex}}_{\mathrm{e}} = 0.015$. We showed that this value of the interfacial jump parameter should be used when the interface is located at $y_{\mathrm{M}} = y_\eta$ (see Eq. (13)). Let us now study the variations of the jump parameter as the position of the interface y_{M} varies within the interfacial zone. From Eq. (13), it is found that $\psi^{\mathrm{ex}} \in [-0.06; 0.09]$ as $y_{\mathrm{M}} \in [-0.275; -0.075]$. This shows that the order of magnitude of the variations of ψ^{ex} is the same as its value. This means that the error on the value of the jump parameter should be expected to be at least 100% as the position of the discontinuous interface varies within the interfacial zone (this variation reaches 1000% if the value at $y_{\mathrm{M}} = y_\eta$ is considered as the reference). This explains why large variations of the results at the macroscopic scale are reported in the literature when the interface position is varied [3,13,20,21]. This is corroborated by the fact that the condition (16) is not satisfied in this case. Indeed, in the present example, $|\psi^{\mathrm{ex}}_{\mathrm{e}}| = 0.015$ whereas $|\psi^+ - \psi^-|\delta \simeq 0.15$. The variable part of the surface-excess quantity ψ^{ex} is thus 10 times larger than its intrinsic part. Therefore, ψ^{ex} cannot be considered as an interfacial property. Nevertheless, it must be emphasized that the macroscopic model provides good results whatever the position of the interface y_{M} within the interfacial zone, provided that the value of the jump parameter ψ^{ex} is modified accordingly; this is shown in [6]. However, we consider that the best choice for y_{M} is y_η and that the corresponding value of the jump parameter is $\psi^{\mathrm{ex}}_{\mathrm{e}}$.

4.2. Turbulent flow perpendicular to the porous/fluid interface

4.2.1. Description of the problem

The momentum transfer at a porous/fluid interface is important in many applications. In this study, we consider a porous medium made of channels parallel to the flow as illustrated in Fig. 8(a). The flow in each channel is turbulent and the $k-\epsilon$ turbulence model can be used to characterize the flow at the channel (i.e., microscopic) scale. Here, we study the momentum transfer at the outlet of the porous medium. Our goal is not to make a full description of the flow at the porous/fluid interface and we restrict our analysis to the momentum transfer; the transfer of turbulence characteristics (turbulent kinetic energy and dissipation) is not studied here. This choice is dictated by the fact that the momentum transfer exhibits characteristics that can be considered as interfacial properties.

At the macroscopic scale, the porous/fluid interface is modeled as a discontinuity generally located right at the outlet of the channels. At this discontinuous interface, the pressure is modeled as being discontinuous: this is the *singular pressure drop* that is generally modeled as follows (e.g. [10])

$$p_{\mathrm{t}}|_{y_{\mathrm{M}}^+} - p_{\mathrm{t}}|_{y_{\mathrm{M}}^-} = -\frac{1}{2}\,\rho\, C_{\mathrm{s}} \langle u \rangle^2 \tag{17}$$

where $\langle u \rangle$ is the volume-averaged velocity (in the y-direction), C_{s} is a non-dimensional parameter called the singular pressure drop coefficient and where p_{t} is the total pressure defined by

$$p_{\mathrm{t}} = (\langle p \rangle_{\mathrm{f}} + \rho \langle u \rangle_{\mathrm{f}}^2)$$

where $\langle p \rangle_{\mathrm{f}}$ and $\langle u \rangle_{\mathrm{f}}$ are the intrinsic volume-averaged pressure and velocity, respectively ($\langle \psi \rangle_{\mathrm{f}} = \langle \psi \rangle / \phi$ for any physical quantity ψ).

The boundary condition (17) is associated to the following macroscopic model:

$$-\frac{\mathrm{d}p_{\mathrm{t}}}{\mathrm{d}y} = \begin{cases} \frac{\mu}{K_{\mathrm{p}}} \langle u \rangle + \rho\, C_p \langle u \rangle^2 & \text{for } y < y_{\mathrm{M}} \\ 0 & \text{for } y > y_{\mathrm{M}} \end{cases}$$

where C_p is a friction coefficient that depends on the Reynolds number and on the wall roughness (e.g. [12]).

Good engineering estimates have been established and for this particular geometry, $C_{\mathrm{s}} \simeq 1$ [10]. No dependence of the singular pressure drop coefficient C_{s} is ever reported and this property should therefore be a genuine interfacial property.

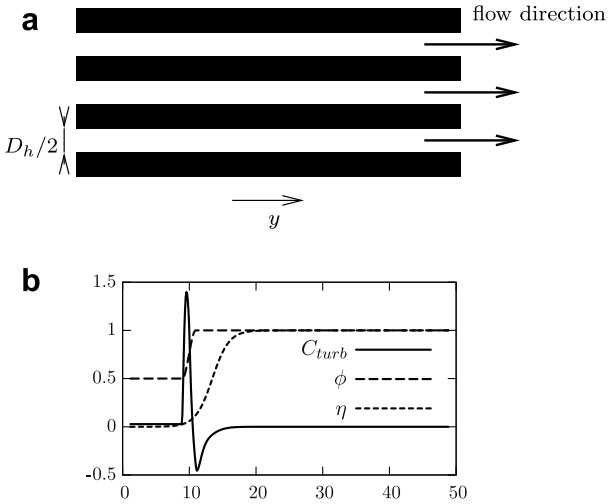

Fig. 8. Turbulent flow at the outlet of a porous medium made of parallel plates. (a) Geometry of the microscopic problem. The $k-\epsilon$ turbulence model is used with $\mathrm{Re_h} = 510^5$. (b) Porosity and turbulent friction coefficient profiles at the mesoscopic scale.

4.2.2. Analysis of the problem using an up-scaling approach

Let us consider a steady-state turbulent flow perpendicular to the porous/fluid interface. Let us assume that this problem can be considered as one-dimensional in the direction y perpendicular to the interface. At the mesoscopic scale, the momentum balance equation obtained by volume-averaging reads:

$$-\frac{\mathrm{d}p_\mathrm{t}}{\mathrm{d}y} = \mu\frac{\langle u\rangle}{K_y(y)} + \rho C(y)u^2 \tag{18}$$

where $K_y(y)$ is the permeability and $C(y)$ is a friction coefficient (e.g. [12]).

Eq. (18) can be made non-dimensional by using the hydraulic diameter of the channels D_h as the characteristic length. One finds:

$$\underbrace{-\frac{D_\mathrm{h}}{\rho\langle u\rangle^2}\frac{\mathrm{d}p_\mathrm{t}}{\mathrm{d}y}}_{C_\mathrm{tot}} = \underbrace{\frac{1}{\mathrm{Re}_\mathrm{h}}\frac{D_\mathrm{h}^2}{K_y}}_{C_\mathrm{lam}} + \underbrace{C D_\mathrm{h}}_{C_\mathrm{turb}} \tag{19}$$

where Re_h is the Reynolds number

$$\mathrm{Re}_\mathrm{h} = \frac{\rho u D_\mathrm{h}}{\mu}$$

The profile of C_lam can be deduced from a laminar simulation and the profile of $(\mathrm{d}p_\mathrm{t}/\mathrm{d}y)$ is easily accessible from a simulation at the microscopic scale of the turbulent flow. From Eq. (19), it is then straigthforward to deduce the profile of C_turb as shown in Fig. 8(b) (the cellular filter has been used [18]).

For the case considered, $C_\mathrm{lam} \ll C_\mathrm{turb}$ so that Eq. (19) can be approximated by

$$-\frac{D_\mathrm{h}}{\rho\langle u\rangle^2}\frac{\mathrm{d}p_\mathrm{t}}{\mathrm{d}y} = C_\mathrm{turb}(y) \tag{20}$$

This first order differential equation is valid at the mesoscopic scale where the porous/fluid "interface" is a continuous transition zone. The determination of the boundary condition at the equivalent discontinuous interface (macroscopic scale) can be deduced from the line of reasoning presented in Section 2.1. It is found that the following boundary condition holds:

$$p_\mathrm{t}|_{y_\mathrm{M}^+} - p_\mathrm{t}|_{y_\mathrm{M}^-} = -\rho\langle u\rangle^2 C_\mathrm{turb}^\mathrm{ex}(y_\mathrm{M}/D_\mathrm{h}) \tag{21}$$

The boundary condition (21) is *exactly* the same as the classical *singular pressure drop* condition (17). Within the framework of the present up-scaling approach, the classical singular pressure drop coefficient C_s is therefore interpreted as the surface-excess friction coefficient:

$$C_\mathrm{s} = 2C_\mathrm{turb}^\mathrm{ex} \tag{22}$$

However, the surface-excess friction coefficient $C_\mathrm{turb}^\mathrm{ex}$ a priori depends on the location of the discontinuous interface y_M. Qualitatively, Fig. 8(b) shows that the profile of C_turb is such that $\max|C_\mathrm{turb} - C_\mathrm{turb}^-| \gg |C_\mathrm{turb}^+ - C_\mathrm{turb}^-|$, so that we expect $C_\mathrm{turb}^\mathrm{ex}$ to depend weakly on y_M.

It is worth emphasizing that the previous qualitative discussion is valid only if the entire variations of C_turb is considered as part of the interfacial phenomena and is thus accounted for in $C_\mathrm{turb}^\mathrm{ex}$. Now, Fig. 8(b) shows that the thicknesses of the profiles $\phi(y)$ and $C_\mathrm{turb}(y)$ are different. Following the discussion of Section 3, we thus introduce a function η whose profile comprises the variations of both ϕ and C_turb. As discussed in Section 3, the position of the equivalent discontinuous interface is then naturally defined by $y_\mathrm{M} = y_\eta$. With the profile of η shown in Fig. 8(b), it is found that $(y_\eta/D_\mathrm{h}) = 13.5$, whereas $(y_\phi/D_\mathrm{h}) = 10$; thus, it is found that $C_{turbe}^\mathrm{ex} = 0.40$ (cf., Eq. (11)). The non-dimensional interfacial thickness is $(\delta/D_\mathrm{h}) \simeq 7$, so that $|(C_\mathrm{turb}^+ - C_\mathrm{turb}^-)(\delta/D_\mathrm{h})| \simeq 0.193$. This value is smaller than that of $C_{turb_\mathrm{e}}^\mathrm{ex}$ and, according to the condition (16), $C_\mathrm{turb}^\mathrm{ex}$ has a rather weak dependence on the position y_M of the interface: $\pm 25\%$ over the interface thickness, a value that must be compared to the $\pm 500\%$ in the case of the laminar flow parallel to the fluid/porous interface. Therefore, $C_\mathrm{turb}^\mathrm{ex}$ can be considered as an intrinsic interfacial property, as expected.

Moreover, if the interface is located right at the outlet of the channels, i.e., $(y_\mathrm{M}/D_\mathrm{h}) = (y_\phi/D_\mathrm{h}) = 10$, one gets $C_\mathrm{turb}^\mathrm{ex} = 0.49$ and the relation (22) is recovered *exactly*. This shows that our up-scaling analysis allows to justify theoretically the classical singular pressure drop "correlation" (17) and to determine the value of the pressure drop coefficient. Furthermore, intuitively, the interfacial zone would be defined rather by the porosity profile than by the profile of η. The thickness of the porosity profile is $(\delta/D_\mathrm{h}) = 2$ and, in this case, the surface-excess friction coefficient $C_\mathrm{turb}^\mathrm{ex}$ (corresponding to the singular pressure drop coefficient C_s) varies only by $\pm 5\%$ as the position of the interface varies over the interface thickness defined by the porosity profile.

4.3. Heat transfer at a fluid/porous interface

Transfer processes at a fluid/porous interface very often involve not only momentum transfer but also heat transfer. Here, we apply our analysis to a steady-state conductive heat transfer problem. In the literature, there is still no agreement on the boundary conditions that should be applied at the fluid/porous interface. Alazmi and Vafai [1] show that all the possible boundary conditions are proposed in the literature: continuity or not of the heat flux combined with continuity or not of the temperature at the interface, all the combinations existing. Thus, this model is viewed as a model problem for which results are available in the literature (e.g. [15,22]).

4.3.1. Boundary conditions at the macroscopic scale

Let us consider a steady-state heat conduction problem at a fluid/porous interface. For the sake of simplicity, we consider that the heat flux is normal to the fluid/porous interface and we consider a one-equation model. Thus, at the mesoscopic scale, we assume that the mean temperature (simply denoted T) satisfies the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(k(y)\frac{\mathrm{d}T}{\mathrm{d}y}\right) = 0 \tag{23}$$

where y is the coordinate normal to the interface and $k(y)$ is the thermal conductivity profile that varies continuously across the interface and that reaches asymptotic values corresponding (i) to the effective thermal conductivity of the homogeneous porous medium, k_p, and (ii) to the thermal conductivity of the fluid, k_f. By integration, one gets:

$$k(y)\frac{\mathrm{d}T}{\mathrm{d}y} = -\varphi \tag{24}$$

where φ is a constant that represents the heat flux across the domain.

At the macroscopic scale, the temperature field is represented by two bulk temperature fields: $\widetilde{T}_p(y)$ in the homogeneous porous medium $y < y_\mathrm{M}$ and $\widetilde{T}_\mathrm{f}(y)$ in the homogeneous free fluid region $y > y_\mathrm{M}$. Within each homogeneous region $i \in \{p; f\}$, the macroscopic heat conduction equation reads

$$k_i\frac{\mathrm{d}\widetilde{T}_i}{\mathrm{d}y} = -\varphi \tag{25}$$

This equation is valid in particular at the discontinuous interface located at y_M. This thus shows that the following boundary condition holds:

$$-k_p \left.\frac{d\widetilde{T}_p}{dy}\right|_{y_M} = -k_f \left.\frac{d\widetilde{T}_f}{dy}\right|_{y_M} = \varphi \tag{26}$$

This equation shows that *the conductive heat flux is continuous at the interface*. Physically, this condition is coherent with the fact that the system is in steady-state: no accumulation of energy can exist anywhere in the system, in particular at the interface.

To determine the boundary condition on the temperature, we follow the line of reasoning presented with Section 2.1 in Eq. (24) that can be rewritten as

$$\frac{dT}{dy} = -\frac{\varphi}{k(y)} \tag{27}$$

It is found that

$$\widetilde{T}_f(y_M) - \widetilde{T}_p(y_M) = -\varphi \left(\frac{1}{k}\right)^{ex}(y_M) \tag{28}$$

Eq. (28) shows that, at the macroscopic scale, *the temperature is a priori discontinuous at the interface*. From a physical point of view, it is easier to rewrite Eq. (28) as follows:

$$\varphi = -\frac{1}{(1/k)^{ex}}(\widetilde{T}_f - \widetilde{T}_p)(y_M) \tag{29}$$

This writing clearly shows that $(1/k)^{ex}$ is a heat resistance, whose value can be *determined* from the profile $k(y)$ only.

Thus, our analysis allows to *determine* (i) the form of the boundary conditions and (ii) the value of the corresponding jump parameter.

4.3.2. Assessment of the proposed model

To assess our analysis, we consider the case of a two-dimensional porous medium made of cubes regularly spaced as shown in Fig. 9(a). This geometry is such that the porosity of the homogeneous porous medium is $\phi = 5/9$. The temperature field at the microscopic scale is computed by imposing constant and uniform temperatures at the lower and upper boundaries of the domain. The thermal conductivity of the solid phase k_s is much larger than that of the fluid phase k_f : $k_s/k_f = 50$. These values have been chosen to be consistent with those considered in [22].

The mesoscopic temperature field $T(y)$ is obtained by volume-averaging the microscopic field using the cellular filter (see Fig. 9(b)). It is important to point out that, unlike in [22], the filter used is identical is the entire domain, i.e., in the homogeneous porous region, in the free fluid region and across the interface. From this temperature field, it is possible to determine the thermal conductivity profile $k(y)$; this is shown in Fig. 9(c).

A first important characteristic of the thermal conductivity profile is that its zone of variation is the same as the zone of variation of the porosity (see Fig. 9(c)). This means that, in this case, the porosity profile is indeed representative of the interfacial zone; thus $\eta = \phi$. Another important characteristic is that the thermal conductivity varies monotonically across the interfacial zone. We thus expect the variable part of $(1/k)^{ex}$ to be larger than its intrinsic part. To confirm this feature, we apply the condition (16). From the profiles $\phi(y)$ and $(1/k)(y)$, it is found that $\delta = 5$, $y_\phi = -0.4$ and $y_{1/k} = -0.5$; thus, the condition (16) is not satisfied.

To assess our analysis further, we compare the mesoscopic temperature profile obtained by volume-averaging to the macroscopic temperature profile obtained by imposing the derived boundary condition (28). We *choose* to locate the interface at the center of gravity of the profile $(1/k)(y)$ where $(1/k)^{ex} = 0$. The motivation for this particular choice is twofold: (i) this particular position is located within the interfacial zone and (ii) at this particular location, the macroscopic temperature is continuous (see Eq. (28)). The macroscopic temperature profile found $\widetilde{T}(y)$ is plotted in Fig. 10(a). This figure shows that the agreement with the mesoscopic temperature profile is excellent. However, if the interface is located at the boundary of the upper row of cubes, i.e., $y_M = -1$, and if the temperature is imposed to be still continuous at this location, the corresponding macroscopic profile is very dif-

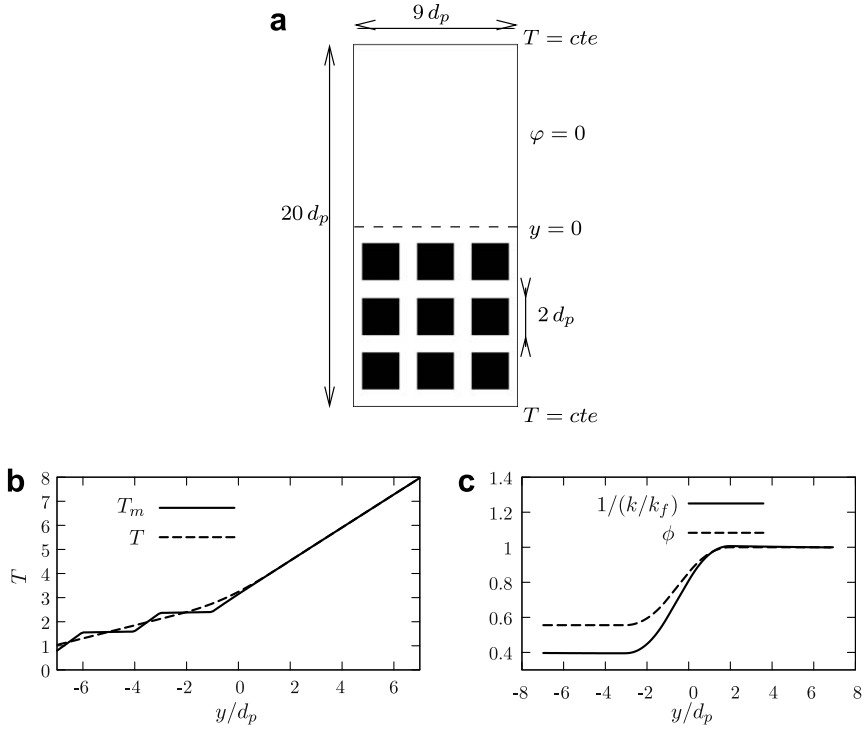

Fig. 9. Conductive heat transfer problem at a fluid/porous interface at the microscopic and mesoscopic scales. (a) Microscopic geometry. (b) Microscopic and mesoscopic temperature profiles both at $x = 0$. (c) Porosity and thermal conductivity profiles.

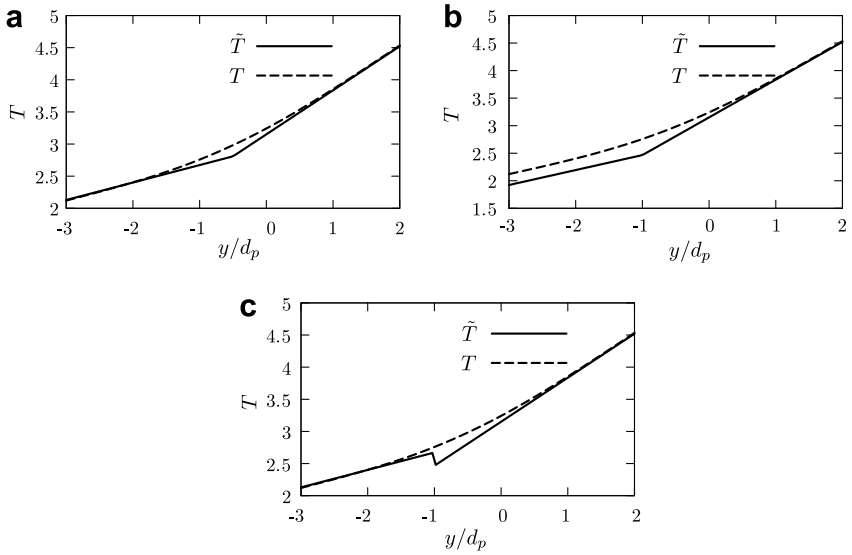

Fig. 10. Macroscopic temperature profile $\tilde{T}(y)$ for different locations y_M of the equivalent discontinuous interface and for different forms of the boundary condition. (a) Interface located at the center of gravity of the profile $(1/k)(y)$. (b) Interface located at the boundary of the upper row of cubes with continuity of the temperature. (c) Interface located at the boundary of the upper row of cubes with temperature jump given by Eq. (28).

ferent from the mesoscopic temperature profile as shown in Fig. 10(b). Nevertheless, if, still at this location, the temperature jump boundary condition (28) is imposed with $(1/k)^{ex} = ((1/k)^{+} - (1/k)^{-})(y_M - y_{1/k})$, the correct macroscopic temperature profile is recovered as shown in Fig. 10(c). These results show the consistence of our approach and analysis.

5. Discussion

It must be emphasized that, in our analysis, the intrinsic part of a surface quantity depends directly on the introduction of the variable η, which is arbitrary. Thus, the value of the intrinsic part is not physically intrinsic because it depends on η that is not a physical variable. Nevertheless, this dependence is rather weak because it depends only on the center of gravity of the profile $\eta(y)$, which is located at about the center of the interfacial zone.

The very reason of this arbitrariness comes from the fact that, in general, if it is possible to define an interfacial thickness with little arbitrariness, it is not possible to exhibit a physical variable that varies monotonically across this interfacial region. It must be noticed that this issue, which exists for the description of transfers at a fluid/porous interface, does not exist in the van der Waals' model of capillarity. Indeed, in this case, all the fluid properties depend on the fluid density ρ that *does* vary monotonically across the liquid/vapor interface. In this case, no arbitrariness exists because the physical variable ρ defines the interfacial region and all the other fluid properties depend on ρ. In the case of a fluid/porous interface, no *physical* variable varies monotonically across the whole interfacial region. This is the very reason why we have introduced an auxiliary function η.

However, it must be emphasized that our analysis allows to put into light the existence of two different parts in a surface-excess quantity: a variable and an intrinsic part. It must be acknowledged that the intrinsic part still exhibits some arbitrariness, but this arbitrariness is limited compared to the common confusion regarding the interpretation of the jump coefficients and in particular whether it should or not be interpreted as an intrinsic quantity. Very often, they are implicitly interpreted as intrinsic quantities, whereas we show that it should not be the case.

At this stage of the analysis, we believe that, for fluid/porous interfaces, it is not possible to avoid the above mentioned arbitrariness. According to our current understanding, the only possibility would be to exhibit a *physical* variable that varies monotonically in the whole interfacial region. However, we have not been able to exhibit such a function.

In this paper, we show that, in general, the value of a jump coefficient, interpreted as a surface-excess quantity, varies with the position of the discontinuous interface where boundary conditions are applied. We show in particular that, if this variable part is not accounted for, the outer solutions are not recovered (see Section 4.3). However, it must be emphasized that, under certain length-scale constraints, i.e., in cases where the outer length is sufficiently large compared to the interfacial zone, the exact location of the interface *might* not be very important to recover a good approximation of the outer solution. To define the associated length-scale constraint, one must compare the variations of the outer solution over the outer length with the variation of the outer solution due to the variation of the jump coefficient. If the former is larger than the latter, then the variable part of the jump coefficient can be neglected. This is illustrated in Fig. 10 on the heat conduction problem. If the outer homogeneous region on the left is very large, the temperature difference between the outer boundary and the interface would be very large. In this case, the temperature jump at the interface is negligible. However, the definition of this constraint depends on the physical problem studied and turns out to be difficult to determine for *any* problem. Moreover, in particularly important problems, such as the Beavers and Joseph experiments [2], the size of the outer domains is such that the corresponding length-scale constraint is not satisfied: the ratio of the outer length and the interfacial thickness varies from 10 to 1. Thus the interface position has a large impact on the outer solutions.

It is worth emphasizing that our analysis is general and can be applied without any length-scale constraint, provided that the variable part of the surface-excess quantity is accounted for.

6. Conclusion

In this paper, we analyze the physical nature of the jump coefficients that appear in the expression for boundary conditions at a fluid/porous interface. Our analysis is based on a two-step up-scaling approach of transport phenomena in the interfacial region. In this

framework, jump coefficients are interpreted as surface-excess quantities. We then show that the value of a surface-excess quantity depends on the position of the interface and that this dependence is linear. This important result explains such a dependence observed in the literature. Despite this dependence, we show that it is possible to decompose a surface-excess quantity into an intrinsic part and a variable part. We then exhibit a simple criterion to determine a priori the intrinsic interfacial nature of a jump coefficient (Eq. (16)). This analysis requires the introduction of a variable, η, whose variation across the interface defines the interfacial zone. The best choice for the position of the discontinuous interface is then the center of gravity of the profile $\eta(y)$.

Our theoretical analysis is successfully assessed on three different problems: laminar and turbulent flows and heat transfer at a fluid/porous interface. In each case, the boundary conditions are derived and the intrinsic nature of the jump coefficients is analyzed.

We believe that the analysis presented in this paper is general enough to be applied to any transport phenomenon at a fluid/porous interface, provided that the effective transport coefficients are known at the mesoscopic scale.

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